KVS BHOPAL REGION CBT TEST AUG 2023
SUBJECT-MATHEMATICS
CLASS-12

TOPICS:
DETERMINANTS
CONTINUITY AND DIFFERENTIABILITY
APPLICATIONS OF DERIVATIVES

Case Study – 1

A function f(x) is said to be continuous in an open interval (a ,b) , if it is continuous at every point in the interval.

A function f(x) is said to be continuous in an closed interval [a ,b] , if f(x) is continuous

in (a, b) and $\lim_{h\to 0} f(a+h) = f(a)$ and $\lim_{h\to 0} f(b-h) = f(b)$.

If a function f(x) = $\begin{cases} \frac{\sin(a+1)x+\sin x}{x} & , \ if \ x < 0 \\ c & , if \ x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & , if \ x > 0 \end{cases}$ is continuous at x= 0 ,

then answer the following questions.

1. The value of a is :

   (a) -3/2 (b) 3/2 (c) 0 (d) -1/2

2. The value of b is :

   (a) 1 (b) -1 (c) 0 (d) Any real number except 0

3. The value of c is :

   (a) 1 (b) 1/2 (c) -1 (d) -1/2

4. The value of ($a^2 + c^2$) is :

   (a) 1/5 (b) 5/2 (c) 2/5 (d) None of these

Feedback

Case - study - I

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

is continuous at $x = 0$

at $x = 0$

LHL $\lim_{x \to 0^-} \left(\dfrac{\sin(a+1)x + \sin x}{x} \right) \Rightarrow \lim_{x \to 0^-} \left[\dfrac{\sin(a+1)x}{x} + \dfrac{\sin x}{x} \right]$

$\lim_{x \to 0^-} \dfrac{\sin(a+1)x}{x} + \lim_{x \to 0^-} \dfrac{\sin x}{x}$

$\lim_{x \to 0^-} \dfrac{\sin x}{x} = 1$

$\lim_{x \to 0^-} \dfrac{(a+1)\sin(a+1)x}{(a+1)(x)} + 1$

$\lim_{x \to 0^-} \dfrac{\sin(a+1)x}{(a+1)x} = 1$

$= a+1+1 \Rightarrow a+2.$ — (1)

RHL $\lim_{x \to 0^+} \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} \Rightarrow \lim_{x \to 0^+} \dfrac{\sqrt{x}\sqrt{(1+bx)} - \sqrt{x}}{bx\sqrt{x}}$

$\lim_{x \to} \dfrac{\sqrt{x}\left[\left(\sqrt{1+bx}\right) - 1\right]}{\sqrt{x}\, bx} \Rightarrow \lim_{x \to 0^+} \dfrac{\sqrt{1+bx} - 1}{bx}$

$\lim_{x \to 0^+} \dfrac{\sqrt{1+bx} - 1}{bx} \times \dfrac{\sqrt{1+bx} + 1}{\sqrt{1+bx} + 1} \Rightarrow \lim_{x \to 0^+} \dfrac{(1+bx) - 1}{bx\left(\sqrt{1+bx} + 1\right)}$

$\lim_{x \to 0} \dfrac{bx}{bx\left(\sqrt{1+bx} + 1\right)} \Rightarrow \dfrac{1}{2}$ — (2)

Limit value. $\lim_{x \to 0} f(x) \Rightarrow \lim_{x \to 0} c = c$ — (3)

LHL = RHL = Limit value.

$a+2 = \frac{1}{2} = c$

$a+2 = \frac{1}{2}$.

(1) $a = \frac{1}{2} - 2$.

$\boxed{a = -\frac{3}{2}}$

(2) b = any Real Number except 0

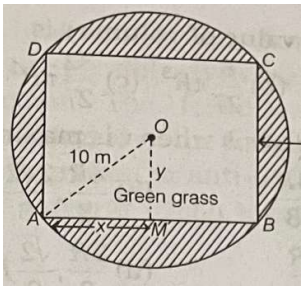
(3) $c = \frac{1}{2}$

(4) $a^2 + c^2 \Rightarrow \frac{9}{4} + \frac{1}{4} \Rightarrow \frac{10}{4} \Rightarrow \frac{5}{2}$

Case - study - (2)

Case Study 2

An architect designs a garden in a society. The garden is in the shape of a rectangle inscribed in a circle of radius 10m as shown in given figure.



Gravelling Path

Based on the above information, answer the following: (Q.5 to Q.8)

Q5. 2x and 2y represents the length and breadth of the rectangular part, then the relation between the variables is

(a) $x^2 - y^2 = 10$ (b) $x^2 + y^2 = 10$ (c) $x^2 - y^2 = 100$ (d) $x^2 + y^2 = 100$

Q6. The area of the green grass A expressed as a function of x is

(a) $2x\sqrt{(100 - x^2)}$ (b) $4x\sqrt{(100 - x^2)}$ (c) $2x\sqrt{(100 + x^2)}$ (d) $4x\sqrt{(100 + x^2)}$

Q7. The value of length of rectangle, if A is maximum, is

(a) $10\sqrt{2}$ m (b) $20\sqrt{2}$ m (c) 20 m (d) $5\sqrt{2}$ m

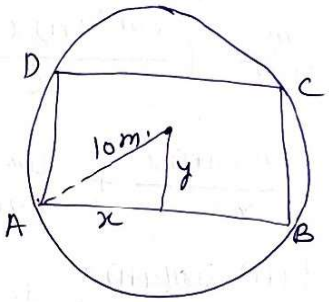
Q8. The area of gravelling path is

(a) $100(\pi + 2)m^2$ (b) $100(\pi - 2)\ m^2$ (c) $200(\pi + 2)\ m^2$ (d) $200(\pi - 2)m^2$

Feedback

Case-study-(2)

Given $AB = 2x$, $BC = 2y$



(5) Relation b/w variables.

$x^2 + y^2 = 100$ (d)

(6) Area of garden. $A = 2x \times 2y$

$= 4xy \Rightarrow 4x\sqrt{100 - x^2}$ (b)

(7) $A^2 = S$

$A(x) = 4x\sqrt{100 - x^2} \Rightarrow A^2 = 16x^2(100 - x^2) = S(x)$

$S'(x) = 16(100x^2 - x^4)$

$S'(x) = 16(200x - 4x^3) \Rightarrow S'(x) = 0$

$S''(x) = 16(200 - 12x^2)$

$200x - 4x^3 = 0$

$x = 0, \quad x = \sqrt{50}$

$S''(5\sqrt{2}) = 16(200 - 12 \times 50) < 0$

$x = 5\sqrt{2}$

length value of Rectangle $2x \Rightarrow 2 \times 5\sqrt{2}$

$= \boxed{10\sqrt{2}}$

(8) When $x = 5\sqrt{2}$

$y = 5\sqrt{2}$

Area of gravelling path $= \pi r^2 - 4xy \Rightarrow \pi(10)^2 - 4(5\sqrt{2})(5\sqrt{2})$

$\Rightarrow 100\pi - 200 \Rightarrow 100(\pi - 2)\ m^2$. (B)

Q9. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

Assertion (A): The function $y = [x(x - 2)]^2$ is increasing in $(0, 1) \cup (2, \infty)$

Reason (R): $dy/dx = 0$, when $x = 0, 1, 2$

(a) Both A and R are true and R is the correct explanation of A

(b)Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false and R is True

Feedback

(9) $y = [x(x-2)]^2$

$y = (x^2 - 2x)^2$

$y = x^4 + 4x^2 - 4x^3$

d.w.r to x

$\frac{dy}{dx} = 4x^3 + 8x - 12x^2$

$\frac{dy}{dx} = 4x(x^2 + 2 - 3x)$

$= 4x(x^2 - 3x + 2)$

$= 4x(x-1)(x-2)$

$\frac{dy}{dx} = 0 \qquad x = 0, 1, 2$

(0,1) f is ↑ increasing fun (1)

$f'(x) > 0$

(2,∞) f is ↑ increasing fun (2)

$f(x) > 0$

$\frac{dy}{dx} = 0$

$x = 0, 1, 2$

Both A and R are true and R is the correct explanation of A.

(A)

Q10 Assertion (A) For two matrices A and B of order 3, |A |=2| B| = -3 then |2AB | is -48.

Reason(R) For a square matrix A, A (adj A) = (adj A) A= |A |I

A Both A and R are true and R is the correct explanation of A

B Both A and R are true but R is NOT the correct explanation of A.

C A is true but R is false

D A is false but R is true

E Both A and R are false

Feedback

(10) $A = [\ ]_{3\times3}$, $B = [\ ]_{3\times3}$

$|A| = 2$, $B = -3$

$|2AB| = -48$

$|2AB| = 2^3|AB| = 2^3|A||B|$

$= 8 \times 2 \times (-3)$

$= -48$

$A(\text{adj } A) = (\text{adj } A)A = |A|I$

Both A and R are True and R is ~~not~~ Not correct explanation of A.

(B)

Answer Key

Case Study 1	a	d	b	b
Case Study 2	d	b	a	B
Q9. A	Q10. B			